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Crystal distortion and the two-channel Kondo effect

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Abstract. We study a simple model of the two-channel Kondo effect in a distorted crystal. This model is then used to investigate the interplay of the Kondo and Jahn–Teller effects, and also the Kondo effect in an impure crystal. We find that the Jahn–Teller interaction modifies the characteristic energy scale of the system below which non-Fermi-liquid properties of the model become apparent. The modified energy scale tends to zero as the limit of a purely static Jahn–Teller effect is approached. We find also that the non-Fermi-liquid properties of the quadrupolar Kondo effect are not stable against crystal distortion caused by impurities.

1. Introduction

The general multichannel Kondo model describes a magnetic impurity of spin S in a sea of conduction electrons having spin $\frac{1}{2}$ and n degenerate orbital channels [1]. Such a model was found by Cox [2] also to apply to a non-magnetic rare-earth impurity such as U^{4+} or Ce^{3+} with an orbitally degenerate doublet ground state. In this case, a pseudospin variable associated with quadrupolar deformations of the local orbital plays the part of the impurity spin in the original multichannel Kondo model, and the real magnetic spin of the conduction electrons plays the part of the channel index. Thus we study the Kondo model with $n = 2$ channels. This is known as the quadrupolar Kondo effect.

This realization of the multichannel Kondo model is particularly interesting, as the spin symmetry of conduction electrons ensures channel symmetry in the model. The channel-symmetric two-channel Kondo model has non-Fermi-liquid low-temperature behaviour, shown by the logarithmic divergence of the impurity magnetic susceptibility and specific heat coefficient [1, 3–5].

In this paper we study the interplay of the quadrupolar Kondo effect with local crystal distortion. We consider two possible sources of this distortion: the Jahn–Teller effect [6] and local distortion caused by non-magnetic disorder in the crystal. For an ion subject to the quadrupolar Kondo effect, the orbital degeneracy of the ion realizes the conditions for the Jahn–Teller effect: an orbital degeneracy will split spontaneously on coupling to phonons. This splitting is accompanied by a distortion of the crystal around the ion. In this case, there will be several distortions producing the same splitting of the orbital degeneracy, and the system may tunnel between these equivalent distortions (the dynamic Jahn–Teller effect). There has been discussion [2, 7, 8] of this interplay of the Jahn–Teller and Kondo effects. It is the intention here to provide a simple model of this interplay, and use the technique of Abelian bosonization to calculate the low-temperature properties of this model.

Also, other (non-magnetic) impurities in the crystal will cause internal strains. The effect of strain on a quadrupolar Kondo ion may be described by a term like $H_1 = gs_z$ in the Hamiltonian, where g is linearly related to components of the local strain tensor: i.e., it

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enters the Hamiltonian like a magnetic field term. We investigate the effect of internal strains on the non-Fermi-liquid behaviour of a quadrupolar Kondo impurity, since this behaviour is the main motivation for studying such systems.

2. The model

Cox [9] has shown that many different crystal symmetries are sufficient to produce a quadrupolar Kondo effect in impurity ions (see also Hirst [10] for a review on coupling between a local moment and conduction electrons). Since we are studying the interplay between this effect and local distortions, we choose a simple crystal symmetry with the hope of producing a tractable model. Thus the model that we consider is one of an impurity ion at a site of tetragonal symmetry (lattice lengths $a = b \neq c$). The impurity is taken to be subject to a quadrupolar Kondo effect. The three-dimensional Kondo Hamiltonian may be reduced to a one-dimensional Hamiltonian by partial-wave projection (see appendix A of reference [4] for a review of this), which in the two-channel case is

$$H_{\mathrm{K}} = \sum_{i=1}^{2} \left\{ H_0[\psi_i] + \frac{I}{2}(s_+ J_i^-(0) + \mathrm{HC}) + I s_z J_i^z(0) \right\} \tag{1}$$

where

$$J_i^-(x) = :\psi_{i\downarrow}^\dagger(x)\psi_{i\uparrow}(x): \qquad \text{and} \qquad J_i^z(x) = \frac{1}{2}:\psi_{i\uparrow}^\dagger(x)\psi_{i\uparrow}(x) - \psi_{i\downarrow}^\dagger(x)\psi_{i\downarrow}(x): \tag{2}$$

are the spin currents in the two channels and

$$H_0[\psi_i] = v_{\mathrm{F}} \sum_{\sigma=\uparrow,\downarrow} \int_{-\infty}^{+\infty} \mathrm{d}x \, :\psi_{i,\sigma}^\dagger(x)(-\mathrm{i}\,\partial_x)\psi_{i,\sigma}(x): \tag{3}$$

describes right-moving fermions for $-\infty < x < +\infty$. This is discussed in the review by Cox and Zawadowski [11].

To this is added a part describing local distortion. The Jahn–Teller distortion may be described by a pseudospin $\boldsymbol{\tau}$. In the current case, only the distortion of the four nearest neighbours in the ab-plane is considered, as only this distortion will couple to the orbitals giving rise to the Kondo effect. Thus we may take $\tau_z = \frac{1}{2}$ to correspond to the distorted state with an expansion in the x-direction and contraction in the y-direction. Likewise $\tau_z = -\frac{1}{2}$ corresponds to the state with expansion in the y-direction and contraction in the x-direction.

These distortions are coupled to the pseudospin $\boldsymbol{s}$ in H_{K} via a term

$$H_1 = \bar{g} s_z \tau_z \tag{4}$$

describing the splitting of the degenerate orbitals by the Jahn–Teller distortion. A further term

$$H_2 = \bar{\Delta}_0 \tau_x \tag{5}$$

is added to describe tunnelling between the states $\tau_z = \pm\frac{1}{2}$. Alternatively, one can say that coupling is restricted to the Q_2-mode (see for example Sturge [12]) described by τ_z and equation (4). Equation (5) contains the effect of warping energy and anharmonicities.

The total Hamiltonian is then

$$H = H_{\mathrm{K}} + H_1 + H_2. \tag{6}$$

Note that to describe the effect of local strain caused by non-Kondo impurities, we set $\bar{\Delta}_0 = 0$. $\bar{g}$ is then linearly related to components of the local strain tensor.

We now follow Emery and Kivelson [5] and use Abelian bosonization and refermionization in the analogue of the Tolouse limit for the one-channel Kondo model to transform our Hamiltonian (6) to a more tractable form. Bosonizing via

$$\psi_{i,\sigma}(x) = \frac{1}{\sqrt{2\pi a_0}} \mathrm{e}^{\mathrm{i}\sqrt{4\pi}\phi_{i\sigma}(x)} \tag{7}$$

and introducing the spin and charge combinations of the fields

$$\begin{aligned} \phi_{\mathrm{c}i}(x) &= \frac{1}{\sqrt{2}}[\phi_{i\uparrow}(x) + \phi_{i\downarrow}(x)] \\ \phi_i(x) &= \frac{1}{\sqrt{2}}[\phi_{i\uparrow}(x) - \phi_{i\downarrow}(x)] \end{aligned} \tag{8}$$

and the spin and spin-flavour combinations of the spin fields

$$\begin{aligned} \phi_{\mathrm{s}}(x) &= \frac{1}{\sqrt{2}}[\phi_1(x) + \phi_2(x)] \\ \phi_{\mathrm{sf}}(x) &= \frac{1}{\sqrt{2}}[\phi_1(x) - \phi_2(x)] \end{aligned} \tag{9}$$

we obtain

$$H_{\mathrm{K}} = H_0[\phi_{\mathrm{s}}] + H_0[\phi_{\mathrm{sf}}] + \frac{I}{2\pi a_0}(s_+ \mathrm{e}^{\mathrm{i}\sqrt{4\pi}\phi_{\mathrm{s}}(0)} \cos[\sqrt{4\pi}\phi_{\mathrm{sf}}(0)] + \mathrm{HC}) + \frac{I}{\sqrt{\pi}} s_z\, \partial_x \phi_{\mathrm{s}}(0) \tag{10}$$

where we have dropped the charge fields $\phi_{\mathrm{c}i}$ as they decouple from the impurity.

Using the canonical transformation

$$U = \mathrm{e}^{\mathrm{i}\sqrt{4\pi} s_z \phi_{\mathrm{s}}(0)} \tag{11}$$

we have

$$H_{\mathrm{K}} \longrightarrow U^\dagger H_{\mathrm{K}} U = H_0[\phi_{\mathrm{s}}] + H_0[\phi_{\mathrm{sf}}] + \frac{I}{\pi a_0} s_x \cos[\sqrt{4\pi}\phi_{\mathrm{sf}}(0)] + \frac{\lambda_+}{\sqrt{\pi}} s_z\, \partial_x \phi_{\mathrm{s}}(0) \tag{12}$$

with $\lambda_+ = I - 2\pi v_F$.

We take $\lambda_+ = 0$ in the following, which is analogous to taking the Tolouse limit in the single-channel Kondo problem. Deviations from $\lambda_+ = 0$ are discussed in section 3, and are irrelevant. This has the effect of decoupling the ϕ_{s}-field from the impurity. Defining for notational simplicity

$$\phi(x) = \sqrt{4\pi}\phi_{\mathrm{sf}}(x) \qquad \text{and} \qquad \bar{\lambda} = \frac{I}{\pi a_0}$$

we have the bosonized Hamiltonian

$$H = H_0[\phi] + \bar{\lambda} s_x \cos[\phi(0)] + \bar{g} s_z \tau_z + \bar{\Delta}_0 \tau_x. \tag{13}$$

The next step is to refermionize this Hamiltonian: this is similar to the refermionization procedure used by Moustakas and Fisher [13] in their study of two-level systems in metals. Care must be taken with the fermionic representation of the pseudospins $\boldsymbol{s}$, $\boldsymbol{\tau}$ in order to ensure that the refermionized Hamiltonian is quadratic and that all of the commutation relations are satisfied. The standard fermionic representation of spins $\boldsymbol{s}_1$, $\boldsymbol{s}_2$ is

$$\begin{aligned} s_{1+} &= d_1^\dagger & s_{1z} &= d_1^\dagger d_1 - \frac{1}{2} \\ s_{2+} &= d_2^\dagger \mathrm{e}^{\mathrm{i}\pi d_1^\dagger d_1} & s_{2z} &= d_2^\dagger d_2 - \frac{1}{2} \end{aligned} \tag{14}$$

and these components are rotated before identifying $\boldsymbol{s}_1 \leftrightarrow \boldsymbol{\tau}$ and $\boldsymbol{s}_2 \leftrightarrow \boldsymbol{s}$, so that

$$\begin{pmatrix} \tau_x \\ \tau_y \\ \tau_z \end{pmatrix} = \begin{pmatrix} s_{1z} \\ s_{1x} \\ s_{1y} \end{pmatrix} \begin{pmatrix} s_x \\ s_y \\ s_z \end{pmatrix} = \begin{pmatrix} s_{2x} \\ -s_{2z} \\ s_{2y} \end{pmatrix}. \tag{15}$$

Then refermionizing ϕ via

$$\mathrm{e}^{\mathrm{i}\phi(x)} = \sqrt{2\pi a_0}\mathrm{e}^{\mathrm{i}\pi(d_1^\dagger d_1 + d_2^\dagger d_2)}\psi(x) \tag{16}$$

gives the quadratic Hamiltonian

$$H = H_0[\psi] + \frac{\bar{\lambda}}{4}\sqrt{2\pi a_0}(d_2^\dagger - d_2)[\psi^\dagger(0) + \psi(0)] + \frac{\bar{g}}{4}(d_2^\dagger - d_2)(d_1^\dagger + d_1) + \bar{\Delta}_0\left(d_1^\dagger d_1 - \frac{1}{2}\right) \tag{17}$$

where $H_0[\psi]$ describes free right-moving fermions for $-\infty < x < +\infty$.

The factor $\exp[\mathrm{i}\pi(d_1^\dagger d_1 + d_2^\dagger d_2)]$ introduced in equation (16) serves to ensure that the commutation relations of the spins with the electron fields are satisfied. It is precisely the Klein factor discussed by Kotliar and Si [14] and in the review by von Delft and Schoeller [15].

Transforming to Majorana fermions via

$$\begin{aligned} \psi(x) &= \frac{1}{\sqrt{2}}[\hat{\chi}(x) + \mathrm{i}\hat{\xi}(x)] \\ d_1 &= \frac{1}{\sqrt{2}}(\hat{b} + \mathrm{i}\hat{c}) \\ d_2 &= \frac{1}{\sqrt{2}}(\hat{\eta} + \mathrm{i}\hat{a}) \end{aligned} \tag{18}$$

the $\hat{\xi}$-field decouples from the impurity, leaving

$$H = H_0[\hat{\chi}] + \mathrm{i}\lambda\hat{a}\hat{\chi}(0) + \mathrm{i}g\hat{a}\hat{b} + \mathrm{i}\Delta_0\hat{b}\hat{c} \tag{19}$$

where

$$\lambda = -\sqrt{\frac{\pi a_0}{2}}\bar{\lambda} \qquad g = -\frac{1}{2}\bar{g} \qquad \Delta_0 = \bar{\Delta}_0.$$

Impurity Green's functions, defined as

$$D_{ij}(t) = -\mathrm{i}\langle T\zeta_i(t)\zeta_j(0)\rangle \tag{20}$$

where $\boldsymbol{\zeta} = (\hat{a}, \hat{b}, \hat{c})$, can be calculated from the Hamiltonian (19) by e.g. the equation-of-motion method. In the Matsubara formulation we obtain

$$\begin{aligned} D_{ij}(\mathrm{i}\omega_n) = \frac{1}{\omega_n^2 + \Delta_0^2} &\frac{1}{\mathrm{i}\omega_n[(\omega_n^2 + g^2 + \Delta_0^2)/(\omega_n^2 + \Delta_0^2)] - \lambda^2 G^{(0)}(\mathrm{i}\omega_n)} \\ &\times \begin{bmatrix} \omega_n^2 + \Delta_0^2 & g\omega_n & g\Delta_0 \\ -g\omega_n & -\mathrm{i}\omega_n[\mathrm{i}\omega_n - \lambda^2 G^{(0)}(\mathrm{i}\omega_n)] & -\mathrm{i}\Delta_0[\mathrm{i}\omega_n - \lambda^2 G^{(0)}(\mathrm{i}\omega_n)] \\ g\Delta_0 & \mathrm{i}\Delta_0[\mathrm{i}\omega_n - \lambda^2 G^{(0)}(\mathrm{i}\omega_n)] & \omega_n^2 + g^2 + \mathrm{i}\omega_n\lambda^2 G^{(0)}(\mathrm{i}\omega_n) \end{bmatrix} \end{aligned} \tag{21}$$

where

$$G^{(0)}(\mathrm{i}\omega_n) = -\frac{\mathrm{i}}{2v_\mathrm{F}}\operatorname{sgn}\omega_n \tag{22}$$

is the free-conduction-electron Green's function.

The impurity contribution to the free energy F may be calculated by evaluating the thermodynamic average of $\hat{a}\hat{\chi}(0)$ using (21) and integrating over the coupling constant:

$$\delta F = \mathrm{i}\lambda \int_0^1 \mathrm{d}\alpha \ \langle \hat{a}\hat{\chi}(0)\rangle_\alpha$$
$$= -\frac{1}{2}T\sum_{\omega_n}\int_0^1 \mathrm{d}\alpha \ \frac{\Gamma \operatorname{sgn}\omega_n}{\omega_n[(\omega_n^2+g^2+\Delta_0^2)/(\omega_n^2+\Delta_0^2)]+\alpha\Gamma\operatorname{sgn}\omega_n} \tag{23}$$

where

$$\Gamma = \frac{\lambda^2}{2v_\mathrm{F}}.$$

The sum over ω_n may be changed to an integral in a standard manner:

$$F = F_0 + \int_{-\Omega}^{\Omega}\frac{\mathrm{d}\omega}{2\pi}\ f(\omega)\tan^{-1}\left(\frac{\Gamma}{\omega\{(-\omega^2+g^2+\Delta_0^2)/(-\omega^2+\Delta_0^2)\}}\right) \tag{24}$$

where F_0 is the free energy of the impurity decoupled from conduction electrons. It is necessary to keep a momentum cut-off Ω to obtain a finite expression for F, but Ω may be taken to infinity in calculating quantities obtained by differentiating F.

3. The Jahn–Teller distortion

The properties of model (19) in the case where $\Delta_0 \neq 0$ (modelling Jahn–Teller distortions) are very different from those in the case where $\Delta_0 = 0$ (modelling deformations caused by local strain fields). This section is thus devoted to the case where $\Delta_0 \neq 0$.

From (24) the impurity contribution to the entropy may be obtained:

$$S = -\frac{\partial F}{\partial T} = S_0 - \int_{-\infty}^{+\infty}\frac{\mathrm{d}\omega}{2\pi}\ \frac{\omega \mathrm{e}^{\omega}}{(\mathrm{e}^{\omega}+1)^2}\tan^{-1}\left(\frac{\Gamma}{T\omega\{(-T^2\omega^2+g^2+\Delta_0^2)/(-T^2\omega^2+\Delta_0^2)\}}\right)$$
$$= S_0 - \delta S \tag{25}$$

where S_0 is the entropy of the impurity decoupled from conduction electrons. This can easily be calculated as

$$S_0 = \ln(2) + \ln(\mathrm{e}^{-E_+/T}+\mathrm{e}^{-E_-/T}) + \frac{1}{T}\frac{E_+\mathrm{e}^{-E_+/T}+E_-\mathrm{e}^{-E_-/T}}{\mathrm{e}^{-E_+/T}+\mathrm{e}^{-E_-/T}} \tag{26}$$

where $E_\pm = \pm\sqrt{g^2+\Delta_0^2}$, which has the low-temperature expansion

$$S_0 = \ln(2) + \left(\frac{2E_+}{T}+1\right)\mathrm{e}^{-2E_+/T} + \mathrm{O}\left(\frac{1}{T}\mathrm{e}^{-4E_+/T}\right). \tag{27}$$

δS may be expanded at low temperature by using

$$\tan^{-1}\left(\frac{\Gamma}{T\omega\{(-T^2\omega^2+g^2+\Delta_0^2)/(-T^2\omega^2+\Delta_0^2)\}}\right)$$
$$= \frac{\pi}{2}\operatorname{sgn}\omega - \tan^{-1}\left(\frac{T\omega}{\Gamma}\frac{g^2+\Delta_0^2}{\Delta_0^2}\right) + \mathrm{O}((T\omega)^3) \tag{28}$$

to obtain

$$S = \frac{1}{2}\ln(2) + \frac{\pi T}{6\Gamma}\frac{g^2+\Delta_0^2}{\Delta_0^2} + \mathrm{O}(T^2). \tag{29}$$

Equation (21) may also be used to examine the local impurity susceptibility

$$\chi_l(T) = \int_0^\beta \mathrm{d}\tau \ \langle\langle s_z(\tau) s_z(0)\rangle\rangle \tag{30}$$

i.e. the susceptibility to a strain field acting on the impurity only. From the refermionization prescription in equations (14) and (18), we have

$$s_z = \mathrm{i}\sqrt{2}\,\hat{a}\hat{b}\hat{c} \tag{31}$$

and thus $\langle\langle s_z(\tau)s_z(0)\rangle\rangle$ must be expanded using Wick's theorem:

$$\begin{aligned}\langle\langle s_z(\tau)s_z(0)\rangle\rangle = -2[&D_{ab}(0)D_{ca}(\tau)D_{bc}(0) - D_{ab}(0)D_{cb}(\tau)D_{ac}(0)\\ &+ D_{ab}(0)D_{cc}(\tau)D_{ab}(0) - D_{ac}(0)D_{ba}(\tau)D_{bc}(0)\\ &+ D_{ac}(0)D_{bb}(\tau)D_{ac}(0) - D_{ac}(0)D_{bc}(\tau)D_{ab}(0)\\ &+ D_{aa}(\tau)D_{bc}(0)D_{bc}(0) - D_{aa}(\tau)D_{bb}(\tau)D_{cc}(\tau)\\ &+ D_{aa}(\tau)D_{bc}(\tau)D_{cb}(\tau) - D_{ab}(\tau)D_{bc}(0)D_{ac}(0)\\ &+ D_{ab}(\tau)D_{ba}(\tau)D_{cc}(\tau) - D_{ab}(\tau)D_{bc}(\tau)D_{ca}(\tau)\\ &+ D_{ac}(\tau)D_{bc}(0)D_{ab}(0) - D_{ac}(\tau)D_{ba}(\tau)D_{cb}(\tau)\\ &+ D_{ac}(\tau)D_{bb}(\tau)D_{ca}(\tau)].\end{aligned} \tag{32}$$

Since we are interested in the low-temperature properties of the model, we find the major contributions to this by finding the discontinuous Green's functions, equation (21). At zero temperature, a discontinuous $D_{ij}(\mathrm{i}\omega_n)$ will give a $D_{ij}(\tau)$ behaving as $1/\tau$ at large τ, whereas a continuous $D_{ij}(\mathrm{i}\omega_n)$ will give a $D_{ij}(\tau)$ behaving at worst as $1/\tau^2$. We find that D_{aa}, $D_{ac} = D_{ca}$ and D_{cc} are discontinuous in the ω-representation. Then defining the constants

$$\begin{aligned}A_1 &= D_{ab}(0)\\ A_2 &= D_{ac}(0)\\ A_3 &= D_{bc}(0)\end{aligned} \tag{33}$$

we find

$$\langle\langle s_z(\tau)s_z(0)\rangle\rangle \simeq -2[A_3^2 D_{aa}(\tau) + 2A_1A_3D_{ca}(\tau) + A_1^2 D_{cc}(\tau)]. \tag{34}$$

These terms arise from terms 1, 3, 7 and 13 of equation (32).

We obtain the large-τ properties of this by examining the small-ω properties of the $D_{ij}(\mathrm{i}\omega_n)$. Since

$$\begin{aligned}D_{aa}(\mathrm{i}\omega_n) &= \frac{1}{\mathrm{i}\Gamma}\,\mathrm{sgn}\,\omega + \mathrm{O}(\omega)\\ D_{ac}(\mathrm{i}\omega_n) &= \frac{g}{\mathrm{i}\Gamma\Delta_0}\,\mathrm{sgn}\,\omega + \mathrm{O}(\omega)\\ D_{cc}(\mathrm{i}\omega_n) &= \frac{g^2}{\mathrm{i}\Gamma\Delta_0^2}\,\mathrm{sgn}\,\omega + \mathrm{O}(\omega)\end{aligned} \tag{35}$$

we have at low temperature and large τ

$$\begin{aligned}D_{aa}(\mathrm{i}\omega_n) &\sim -\frac{1}{\pi\Gamma\tau}\\ D_{ac}(\mathrm{i}\omega_n) &\sim -\frac{g}{\pi\Gamma\Delta_0\tau}\\ D_{cc}(\mathrm{i}\omega_n) &\sim -\frac{g^2}{\pi\Gamma\Delta_0^2\tau}.\end{aligned} \tag{36}$$

Thus

$$\chi_l(T) \simeq \frac{2}{\pi\Gamma}\left(A_3 + \frac{g}{\Delta_0}A_1\right)^2 \int_{\tau_0}^{1/T} \frac{\mathrm{d}\tau}{\tau} \simeq \frac{2}{\pi\Gamma}\left(A_3 + \frac{g}{\Delta_0}A_1\right)^2 \ln\left(\frac{1}{T\tau_0}\right) \tag{37}$$

where τ_0 is a cut-off chosen such that the asymptotics (36) are applicable (i.e. this result holds for $T \ll \tau_0$).

We may also study deviation from the Tolouse limit—i.e. $\lambda_+ \neq 0$ in equation (12). The operator that we are including is

$$O_+ = \frac{\lambda_+}{\sqrt{\pi}} s_z \, \partial_x \phi_\mathrm{s}(0) \tag{38}$$

and if we refermionize ϕ_s and transform to Majorana fermions via

$$\begin{aligned} \psi_\mathrm{s}(x) &= \frac{1}{\sqrt{2\pi a_0}} \mathrm{e}^{\mathrm{i}\sqrt{4\pi}\phi_\mathrm{s}(x)} \\ \psi_\mathrm{s}(x) &= \frac{1}{\sqrt{2}}[\hat{\chi}_\mathrm{s}(x) + \mathrm{i}\hat{\xi}_\mathrm{s}(x)] \end{aligned} \tag{39}$$

we find the following expression for this operator:

$$O_+(\tau) = -\hat{a}(\tau)\hat{b}(\tau)\hat{c}(\tau)\hat{\chi}_\mathrm{s}(0,\tau)\hat{\xi}_\mathrm{s}(0,\tau). \tag{40}$$

At the Emery–Kivelson line, the impurity spin s_z is decoupled from the ψ_s-electrons, so the O_+–O_+ correlation function factorizes into the spin–spin correlation and the density–density correlation function for free electrons:

$$\langle\langle O_+(\tau)O_+(0)\rangle\rangle = \langle\langle s_z(\tau)s_z(0)\rangle\rangle \left[G^{(0)}(\tau)\right]^2. \tag{41}$$

Since

$$\begin{aligned} G^{(0)}(\tau) &\sim 1/\tau \\ \langle\langle s_z(\tau)s_z(0)\rangle\rangle &\sim 1/\tau \end{aligned} \tag{42}$$

at large τ and zero temperature, we have

$$\langle\langle O_+(\tau)O_+(0)\rangle\rangle \sim 1/\tau^3 \tag{43}$$

and thus the operator O_+ has scaling dimension 3/2 and is irrelevant.

However, as found by Sengupta and Georges [17], it is this operator which is responsible for the behaviour of the impurity specific heat. Comparison with the calculation in reference [16] gives the result (again for $T \ll \tau_0$)

$$\frac{\partial C_\mathrm{imp}(T)}{\partial T} \simeq \frac{\lambda_+^2}{2\pi\Gamma v_\mathrm{F}^2}\left(A_3 + \frac{g}{\Delta_0}A_1\right)^2 \ln\left(\frac{1}{T\tau_0}\right). \tag{44}$$

Comparing equation (29) with the $T \to 0$ entropy of the two-channel Kondo model [3, 16]

$$S(0) = \frac{1}{2}\ln 2 \tag{45}$$

we see that the Jahn–Teller interaction does not change the residual entropy of the two-channel Kondo model. This residual entropy is an important and peculiar feature of non-Fermi-liquid two-channel Kondo models. Since it persists in the presence of a Jahn–Teller interaction, it is clear that the non-Fermi-liquid behaviour also persists.

The results [16] for the two-channel Kondo local impurity susceptibility

$$\chi_l^\mathrm{K}(T) \simeq \frac{1}{\pi\Gamma}\ln\left(\frac{\Gamma}{T}\right) \tag{46}$$

and specific heat coefficient

$$\frac{\partial C_{\mathrm{imp}}^{\mathrm{K}}(T)}{\partial T} \simeq \frac{\lambda_+^2}{8\pi\Gamma v_{\mathrm{F}}^2} \ln\left(\frac{\Gamma}{T}\right) \tag{47}$$

show that the logarithmic divergence of these quantities is governed by the characteristic energy scale of the system, Γ. Comparison with equations (37) and (44) shows that this divergence persists but with a modification of the characteristic energy scale to τ_0^{-1}. We find, as $\Delta_0 \to 0$,

$$\tau_0 \sim \frac{g^2 + \Delta_0^2}{\Gamma \Delta_0^2} \tag{48}$$

and thus the non-Fermi-liquid behaviour emerges only at lower and lower temperatures. This is because the *dynamic* nature of the Jahn–Teller effect preserves symmetry of the system, whereas a purely static Jahn–Teller effect—identical to local distortion of the crystal—breaks the symmetry of the system and destroys the non-Fermi-liquid behaviour.

Thus the Jahn–Teller effect does not destroy the quadrupolar Kondo effect, but modifies the characteristic energy scale of the system.

4. Crystal distortion by impurities

Setting $\Delta_0 = 0$ in (19) leaves us with a model for a quadrupolar Kondo impurity sited in a strain field caused by other impurities in the metal. We wish to obtain properties of this model which have been averaged over all possible distributions of these impurities. Thus the local impurity susceptibility will be given by

$$\langle \chi_l(T) \rangle_g = \int P[g] \chi_l(T, g)\ \mathrm{d}g \tag{49}$$

where $P[g]$ is the probability distribution of the local strain field, and $\chi_l(T, g)$ the impurity susceptibility of the two-channel Kondo model in a field, which may be calculated from (24).

The quantity $P[g]$ is calculated in references [18] and [19]. It is Lorentzian at small g and small impurity concentration, with Gaussian tails at high g. For calculational simplicity, we use

$$P[g] = \frac{a}{\pi} \frac{1}{g^2 + a^2} \tag{50}$$

i.e. a Lorentzian distribution for all g. This is possible as impurities in large strain fields have very low susceptibility, so the large-g behaviour of $P[g]$ is unimportant.

$\chi_l(T, g)$ has the properties that at $g = 0$, it is logarithmically divergent as $T \to 0$, but this logarithmic divergence is rounded off at finite g, and $\chi_l(T, g \neq 0) \to$ constant as $T \to 0$. Thus we may use

$$\chi_l(T, g) = \ln(T^2 + g^2) \tag{51}$$

to obtain qualitative results on the behaviour of $\langle \chi_l(T) \rangle_g$.

These approximations give the result

$$\langle \chi_l(T) \rangle_g \simeq \frac{2\pi}{a} \ln(|a| + T) \tag{52}$$

i.e. the impurity susceptibility is finite as $T \to 0$. Thus the non-Fermi-liquid properties of the quadrupolar Kondo effect are not stable against crystal distortion caused by impurities.

The effect of impurities on the low-temperature behaviour of the impurity susceptibility was discussed by Dobrosavljević *et al* [20]. They considered the effect of local density-of-states fluctuations caused by the disorder on the susceptibility. They found that these fluctuations *enhanced* the non-Fermi-liquid properties of the model.

The effect that we considered above suppresses the non-Fermi-liquid behaviour, and is thus in competition with that considered by Dobrosavljević *et al*.

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